A Non-supersymmetric Deformation of the AdS/CFT Correspondence

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ABSTRACT

We deform the AdS/CFT Correspondence by the inclusion of a non-supersymmetric scalar mass operator. We discuss the behaviour of the dual 5 dimensional supergravity field then lift the full solution to 10 dimensions. Brane probing the resulting background reveals a potential consistent with the operator we wished to insert.

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1 Introduction

A corner stone of the AdS/CFT Correspondence [1, 2, 3] is that the supergravity fields in $AdS_5 \times S^5$ act as source terms for operators in the $\mathcal{N}=4$ gauge theory dual. It is essential that there is a one to one mapping between the supergravity fields and these operators. A corollary of this relationship is that we can study any deformation of the $\mathcal{N}=4$ gauge theory since we can introduce all possible operators.

An industry has grown in attempting to generate gravity duals of all interesting deformations of the AdS/CFT Correspondence [4, 5, 6, 7, 8] and understand how different field theory phenomena are encoded gravitationally. Much of the early work in this respect has concentrated on supersymmetric deformations such as the $\mathcal{N}=4$ theory on moduli space [9], $\mathcal{N}=1$ Leigh Strassler theory [10, 11], the $\mathcal{N}=2^*$ [12, 13, 14, 15, 16] and $\mathcal{N}=1^*$ theories [17, 18, 10]. Allowing non-trivial solutions for the supergravity fields is relatively straightforward at the 5d supergravity level. Connecting the resulting backgrounds to the field theory has proven difficult with more success being obtained in the cases where the solutions have been lifted to 10d. The technology to lift solutions to 10d [19, 9, 12, 10] has been developed but in more complicated cases such as the $\mathcal{N}=1^*$ theory [10] the complexity can grow to prohibit a full solution being found. The major benefit of a 10d solution is that brane probing [1, 15, 16, 11, 20, 21, 22] can be used to directly convert the space-time background into a U(1) gauge theory on the surface of the probe. In this way the gauge coupling, operator parmetrization and potentials can be successfully investigated. It should be noted that, since the $\mathcal{N}=4$ gauge theory is strongly coupled in the ultra-violet, fields can never be decoupled from the strong interactions by making them massive in these models.

More recently interest has turned to non-supersymmetric deformations of gauge/gravity duals [6, 23, 24, 25, 26]. It is interesting to see whether the dualities continue to make sense without supersymmetry and there is also the potential to investigate new phenomena not present in supersymmetry. Apart from an early paper on non-supersymmetric deformations in 5d [6], recent attention [23, 24, 25, 26] has focused on deformations of more involved $\mathcal{N}=1$ supersymmetric constructions such as the Maldecena Nunez [27] and the Klebanov Strassler [28] backgrounds. These theories have discrete vacua and hence supersymmetry breaking perturbations will not result in an unstable background. In this paper we return to deforming the orginal AdS/CFT correspondence. We will in fact introduce a mass term of the form $(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2-2\phi_5^2-2\phi_6^2)$ which is naively unbounded. Our interest is in developing the technology to find and lift these solutions to 10d so we will not be so concerned by the runaway behaviour (although the 10d solution we provide correctly reproduces the expected behaviour). One might hope that there would be such backgrounds that are really stable since an $\mathrm{SO}(6)_R$ singlet scalar mass term is not visible in the supergravity solution as it is not in a short multiplet. It's presence could stabilize the solution. Note that the supersymmetric deformations [5, 12, 10] already mentioned require this operator to be present. In fact our brane probe potential reveals the operator not to be present in our 10d lifts. Our solution is also of interest since it is probably the simplest example of a non-supersymmetric deformation; only the metric and four potential fields are non-zero.

In the next section we will discuss the introduction of our deformation at the 5d supergravity level. In section 3 we then lift the full solution to 10d. In section 4 we brane probe the background with a D3 brane and show that asymptotically the background indeed includes the operator we hoped to introduce showing the consistency of the techniques. Finally we plot the potential seen by the probe for the full solution.

2 Deformations in 5d Supergravity

According to the standard AdS/CFT Correspondence map [2, 3] each supergravity field plays the role of a source in the dual field theory. The simplist possibility is to consider non-trivial dynamics for a scalar field in the 5d supergravity theory. We only allow the scalar to vary in the radial direction in AdS with the usual interpretation that this corresponds to renormalization group running of the source. As is standard in the literature [4, 6] we look for solutions where the metric is described by

$$ds^2 = e^{2A(r)} dx^\mu dx_\mu + dr^2 \tag{1}$$

where $\mu = 0..3$ and r is the radial direction in AdS_5. The scalar field has a lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\lambda)^2 + V(\lambda) \tag{2}$$

There are two independent, non-zero, elements of the Einstein tensor (G_{00} and G_{rr}) giving two equations of motion plus there is the usual equation of motion for the scalar field [4]

$$\lambda'' + 4A'\lambda' = \frac{\partial V}{\partial \lambda} \tag{3}$$

$$6A'^2 = \lambda'^2 - 2V \tag{4}$$

$$-3A'' - 6A'^2 = \lambda'^2 + 2V \tag{5}$$

In fact only two of these equations are independent but it will be useful to keep track of all of them.

In the large r limit, where the solution will return to AdS_5 at first order and $\lambda \to 0$ and $V \to m^2\lambda^2$, only the first equation survives with solution

$$\lambda = \mathcal{A} e^{-\Delta r} + \mathcal{B} e^{-(4-\Delta) r} \tag{6}$$

with

$$m^2 = \Delta(\Delta - 4) \tag{7}$$

$\mathcal{A}$ is interpreted as a source for an operator and $\mathcal{B}$ as the vev of that operator since e^r has conformal dimension 1.

If the solution retains some supersymmetry then the potential can be written in terms of a superpotential [7]

$$V = \frac{1}{8}\left|\frac{\partial W}{\partial \lambda}\right|^2 - \frac{1}{3}|W|^2 \tag{8}$$

and the second order equations reduce to first order

$$\lambda' = \frac{1}{2}\frac{\partial W}{\partial \lambda}, \qquad A' = -\frac{1}{3} W \tag{9}$$

2.1 A Scalar Operator

Let us now make a particular choice for the scalar field we will consider. We take a scalar from the multiplet in the 20 of SO(6). These operators have been identified [3] as playing the role of source and vev for the scalar operator $tr\phi_i\phi_j$ in the field theory. In particular we will chose the scalar corresponding to the operator

$$\mathcal{O} = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 - 2\phi_5^2 - 2\phi_6^2 \tag{10}$$

This scalar has been studied in the literature [9, 20] already in its role of describing an $\mathcal{N} = 4$ preserving scalar vev and as a mixture of a mass term and a vev in the $\mathcal{N} = 2^*$ gauge theory [12, 13]. The potential for the scalar, which we will write as $\rho = e^{\lambda/\sqrt{6}}$ is given by

$$V = -\frac{1}{\rho^4} - 2\rho^2 \tag{11}$$

and the three equations of motion become

$$\frac{\rho''}{\rho} - \left(\frac{\rho'}{\rho}\right)^2 + 4\frac{\rho'}{\rho}A' = \frac{\rho}{6}\frac{\partial V}{\partial \rho} \tag{12}$$

$$6A'^2 - 6\left(\frac{\rho'}{\rho}\right)^2 = -2V \tag{13}$$

$$A'' = -4\left(\frac{\rho'}{\rho}\right)^2 \tag{14}$$

The last of these is the sum of (4) and (5). The asymptotic ($r \to \infty$) solutions take the form

$$\lambda = \mathcal{A}e^{-2r} + \mathcal{B}re^{-2r} \tag{15}$$

with $\mathcal{A}$ the scalar vev and $\mathcal{B}$ a mass term for the operator $\mathcal{O}$.

In the special case where only the first part of the solution is present the deformation preserves $\mathcal{N} = 4$ supersymmetry. The superpotential is

$$W = -\frac{1}{\rho^2} - \frac{1}{2}\rho^4 \tag{16}$$

and the second order equations reduce to the first order equations

$$\frac{\partial\rho}{\partial r} = \frac{1}{3}\left(\frac{1}{\rho} - \rho^5\right), \qquad \frac{\partial A}{\partial r} = \frac{2}{3}\left(\frac{1}{\rho^2} + \frac{1}{2}\rho^4\right) \tag{17}$$

with solution [9]

$$e^{2A} = l^2\frac{\rho^4}{\rho^6 - 1} \tag{18}$$

with l^2 a constant of integration.

2.2 Non-supersymmetric First Order Equations

In [29] it was pointed out that using Hamilton Jacobi theory the second order equations could be replaced by a system of first order equations. They further stated that a "superpotential", W, could be found which resulted in the equations (9) even for the non supersymmetric solution with only $\mathcal{B}$ switched on. A similar result was obtained in [30, 31] but as a requirement for the RG flow solution to be stable. Reducing the equations to first order would be very helpful, but the system we discuss here can not be.

Consider the UV of the theory where, expanding (11)

$$V = -3 - 2\lambda^2 + \sqrt{\frac{8}{27}}\lambda^3 + ... \tag{19}$$

we can attempt to find a superpotential W that reproduces this potential via the trial form

$$W = a + b\lambda^2 + c\lambda^3 + ... \tag{20}$$

Working to quadratic order one finds